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Aging dynamics of ±J Edwards-Anderson spin glasses

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Abstract. We analyze by means of extensive computer simulations the out of equilibrium dynamics of Edwards-Anderson spin glasses in $d = 4$ and $d = 6$ dimensions with $\pm J$ interactions. In particular, we focus our analysis on the scaling properties of the two-time autocorrelation function in a wide range of temperatures from $T = 0.07T_c$ to $T = 0.75T_c$ in both systems. In both the $4d$ and $6d$ models at very low temperatures we study the effects of discretization of energy levels. Strong sub-aging behaviors are found. We argue that this is because in the times accessible to our simulations the systems are only able to probe activated dynamics through the lowest discrete energy levels and remain trapped around nearly flat regions of the energy landscape. For temperatures $T \geq 0.5T_c$ in $4d$ and $6d$ we find logarithmic scalings that are compatible with simple dynamical ultrametricity. Nevertheless the behaviour of the systems, even in $6d$ is very different from the mean field SK model results.

PACS. 75.10.Nr Spin-glass and other random models

1 Introduction

After more than twenty years of extensive research, the physics of spin glasses is still far from being completely understood. The inherent complexity of the physical scenario together with unsurmountable mathematical difficulties undermine even the simplest theoretical approaches that attempt to include basic realistic ingredients. Notwithstanding that a large amount of information could be extracted from the analysis of the mean field model (*i.e.* the Sherrington-Kirkpatrick model [1,2]), the extension of those results to finite dimensional spin glasses remains a matter of hard controversy in the statistical physics community [3,4]. In this context large scale numerical simulations emerged as a valuable aid in gaining physical insight into more realistic models [5,6]; however, they pose such a serious demand over existent computational capabilities that the study of the low temperature phases of these systems may be considered in many respects to be still in an exploratory state.

Regarding the out of equilibrium behavior of finite dimensional spin glasses, much attention has been devoted to models with continuous couplings distributions [6,7]. One important feature of these systems is that the ground state is unique. For Gaussian couplings, the aging behavior of models in $3d$ and $4d$ reported in the literature seems to be compatible with the simple aging or weak sub-aging scenarios (to be defined below). These particular scenarios suggest a rather simple phase space structure with a unique relevant time scale, namely the age of the system. This apparently simple behavior could be a consequence of the existence of strongly separated temporal scales. Because of this during the time span of a simulation or experiment at very low temperatures the system could not be able to cross over between different time regimes and only one relevant time scale is probed.

One may wonder whether the relatively simple dynamical behavior observed in spin glasses with continuous distributions at very low temperatures can also be expected in systems with discrete couplings, which present a strong degeneracy of their ground states as well as a noticeable discretization of their low-energy spectrum. That might lead to very low temperature effects that could not be observable in continuous spin glasses.

It is known that at temperatures relatively close to T_c there is no qualitative difference in the phenomenological characterization of aging [10] between the continuous and discrete couplings systems. However, at lower temperatures, effects deriving from the discrete nature of the energy spectrum as well as from the high degeneracy of the ground states may become apparent in discrete models.

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Scaling properties contribute to a quantitative description of complex phenomena, even in cases where a general theory is lacking [3,11,12]. In this respect, different dynamical universality classes may emerge for which similar scaling functions describe the dynamics of different systems. The knowledge of these scaling rules gives considerable insight into the nature of the underlying dynamical processes.

In this paper we present the results of an extensive numerical study of the aging dynamics and scaling properties of the two-time autocorrelation functions for the $\pm J$ Edwards-Anderson spin glasses in dimensions $d = 4$ and $d = 6$ at temperatures which cover the whole low temperature phases of the models.

At the very low temperatures $T = 0.07T_c$ and $T = 0.15T_c$ we find strong sub-aging scenarios both in $4d$ and $6d$. In particular, for the lowest temperature we find a small exponent indicative of a rather quick relaxation to a stationary dynamics. At first sight this seems hard to reconcile with the expected very slow relaxation which takes place at these extremely low temperatures. Nevertheless this can be seen as a consequence of the discrete nature of energy levels which can produce some non trivial time dependent phenomena at very low temperatures at time scales roughly independent of the waiting time t_w. The systems simply do not have enough time to relax over barriers dependent on t_w. So this dynamics is completely different from the usual long time non-equilibrium relaxation but is not trivial due to the discreteness of the low lying energy levels. At $T = 0.5T_c$ we observe logarithmic scalings both in $4d$ and $6d$ which are compatible with dynamic ultrametricity. At this temperature simple aging scalings are also compatible with the numerical data although not as good as logarithmic scalings. As $d = 6$ is the upper critical dimension we expected the relaxational dynamics should present some features of the infinite range SK model. We find insted that the behavior is more similar to that at lower dimensions than to the mean field model.

The paper is organized as follows: in Section 2 we define the model studied and the observables measured; in Section 3 we analyze the results of the $4d$ and $6d$ models respectively. Finally the conclusions and a discussion are presented in Section 4.

2 Model and method

The system consists of a d-dimensional hypercubic lattice of Ising spins which interact according to the following Hamiltonian:

$$\mathcal{H} = -\sum_{\langle i,j \rangle} J_{ij} S_i S_j \ , \qquad (1)$$

where the symbol $\langle i, j \rangle$ indicates that only first neighbor pairs i, j are taken into account. The coupling constants J_{ij} are binary random variables chosen from the following probability distribution:

$$\rho(J_{ij}) = \frac{1}{2}\Big(\delta(J_{ij} - 1) + \delta(J_{ij} + 1)\Big) \ . \qquad (2)$$

The time evolution of the model is governed by a standard heat-bath Monte Carlo process with sequential random update. The imposed boundary conditions were periodic in $4d$ and helical in $6d$ [13]. In the practical implementation of the numerical algorithm a significant increase in speed was accomplished by using multi-spin coding [13] so each spin and coupling constant demand just one bit of information each for storage. In this way we can run many replicas at the same time at the cost of a single realization. In all cases the dynamics is initiated from a random configuration, simulating a sudden quench from infinite temperature into the spin glass phase.

One straightforward way to characterize the out of equilibrium dynamics of complex magnetic systems is through the analysis of the two-time autocorrelation function $C(t, t')$, which can exhibit history dependent features usually referred to as *aging*. A system that has attained thermodynamic equilibrium will show a stationary dynamics for which only time differences make physical sense, and therefore $C(t, t') \equiv C(t-t')$. However, complex magnetic systems such as spin glasses show a much more complex behavior due to the presence of an extremely slow relaxational dynamics. These materials may be out of equilibrium for spans of time longer than any available time scale in the laboratory. In this circumstances insight into the ongoing processes can be obtained by studying the scaling properties of dynamical quantities like $C(t_w + t, t_w)$, where the *waiting time*, t_w, stands for the *age* of the system measured after a quench into the spin glass phase, and t stands for the time measured since the age t_w.

In the numerical experiments we compute the quantity

$$C(t_w + t, t_w) = \left[\frac{1}{N}\sum_{i=1}^{N} S_i(t_w + t) S_i(t_w)\right]_{av} , \qquad (3)$$

where we have denoted by $[\ldots]_{av}$ an average taken over several realizations of the random couplings and thermal histories. An additive form for the autocorrelations was assumed:

$$C(t_w + t, t_w) = C_{st}(t) + C_{ag}\left(\frac{h(t_w + t)}{h(t_w)}\right) \cdot \qquad (4)$$

The stationary part $C_{st}(t)$ is well described by an algebraic decay of the form $C_{st}(t) = At^{-x(T)} + q$. There is no theoretical basis for determining the scaling function $h(z)$ appearing in the aging part of the autocorrelations. Some possibilities have been proposed which describe reasonably well both numerical and experimental data [14]. Experimental data can be accounted for with a scaling function of the form:

$$h(t) = \exp\left[\frac{1}{1-\mu}\left(\frac{t}{\tau}\right)^{1-\mu}\right] \qquad (5)$$

with τ a microscopic time scale. This form can interpolate from the so called *sub-aging* relaxation when $\mu < 1$ to the *super-aging* relaxation when $\mu > 1$ including the

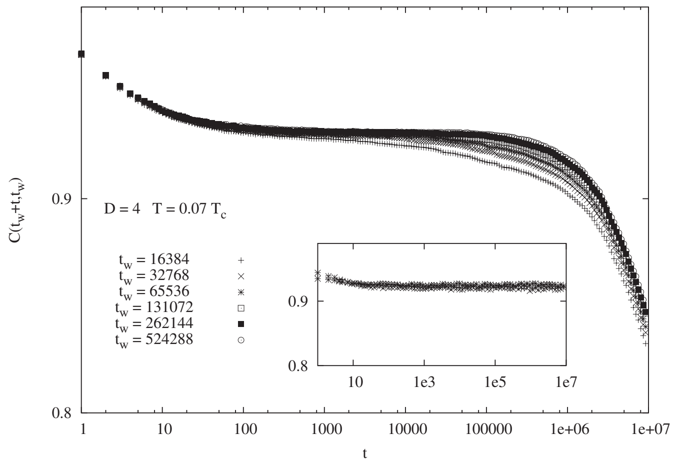

Fig. 1. Two-time autocorrelation function for the $\pm J$ spin glass in $d = 4$ with linear size $L = 12$ after a quench to $T = 0.07T_c$ in a double logarithmic plot. The waiting times are $t_w = 2^k, k = 14 \ldots 19$ from bottom to top. Inset: the result of a microcanonical run after t_w for the three longest t_w' s.

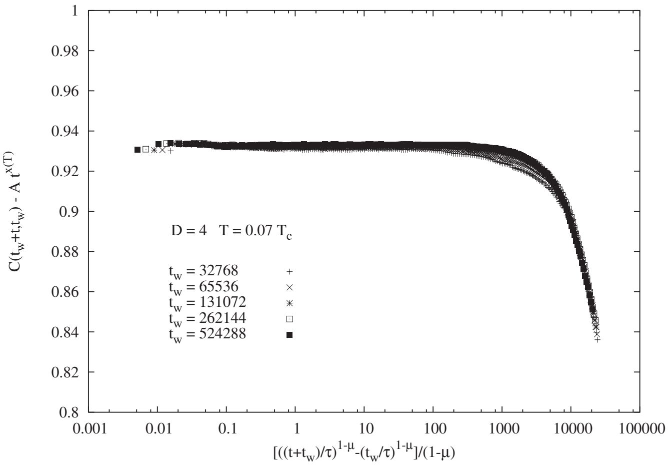

Fig. 2. Sub-aging scaling for the long time regime of the data shown in Figure 1. The stationary decay of the correlations has been subtracted, fit parameters are $A = 0.04$ and $x = 0.57$. The sub-aging exponent is $\mu = 0.4$ and τ has been arbitrarily fixed to one.

commonly observed *simple aging* for $\mu = 1$ [15]. In the case $\mu = 0$ a stationary dynamics is recovered. We will see that this form works well for our data at the lowest temperatures implying a strong sub-aging scenario with an effective relaxation time of the order of t_w^{μ}. Another functional form which has also been used to fit data is the *enhanced power law*:

$$h(t) = \exp\left[(\ln(t/\tau))^{a}\right] \tag{6}$$

which for $a > 1$ implies sub-aging. We have found that these scaling functions work well for the whole range of temperatures studied in both $4d$ and $6d$ although at $T = 0.5T_c$ an ultrametric logarithmic scaling works slightly better.

3 d = 4

The simulations of the four dimensional Edwards-Anderson model were done for systems of linear size $L = 12$ ($T = 0.07T_c$) and $L = 10$ imposing periodic boundary conditions. We recall that the critical temperature for this model has been estimated to be $T_c \approx 2$ [8,9].

3.1 Very low temperatures: probing the discreteness of the lowest energy levels

In Figure 1 we show the behavior of the two-time autocorrelation function $C(t_w + t, t_w)$ at the temperature $T = 0.07T_c$. The waiting times are $t_w = 2^k, k = 14 \ldots 19$, and the simulation was run up to $t = 10^7$ MCS. There is a weak dependence on the waiting time and the system seems to relax to a stationary regime presenting a strong sub-aging behavior.

In Figure 2 we show the best scaling obtained for the long time behavior of the autocorrelation function after subtraction of the stationary part:

$$C_{st}(T) = At^{-x(T)} + q. \tag{7}$$

In this particular case, the best fit was obtained for $x(T) = 0.57$ (a rather high value) and $q \approx 0.93$. In the long time regime, the (weakly) t_w dependence can be well collapsed by the form (5) with a characteristic exponent $\mu = 0.4$. This is a strong sub-aging behaviour. An enhanced power law of the form (6) works also very well with an exponent $a = 9$.

The observation of these sub-aging regimes at this very low temperature is consequence of the discrete nature of the low lying energy levels. In fact the smallest time scale for activation in this energy landscape is of the order of $\exp(2/T) \approx 1,6 \times 10^6$ which is approximately the time where the correlation leaves the quasi-equilibrium regime (see Fig. 1). Up to this time scale the system relaxes effectively in the flat energy landscape defined by the sites with zero local field which are free to flip (besides a smaller group which can still contribute to lowering the energy). Note that the value of the correlation in the plateau $C = q$ in this regime does not correspond to the equilibrium order parameter q_{EA} but instead to one minus the fraction of sites with zero field [23]. After a time of the order of $\exp(2/0.07)$ thermal activation begins to take place through the lowest lying barriers and the correlations decay to zero as the system expands its available phase space. As a further test for this interpretation we have performed a set of simulations in a microcanonical ensemble which are shown in the inset of Figure 1: for the three longest waiting times we have allowed the system to relax up to t_w and from then on only flips of spins with zero local fields were allowed. In these conditions the correlations decayed only to the plateau suggesting that the long time decay in the canonical dynamics was

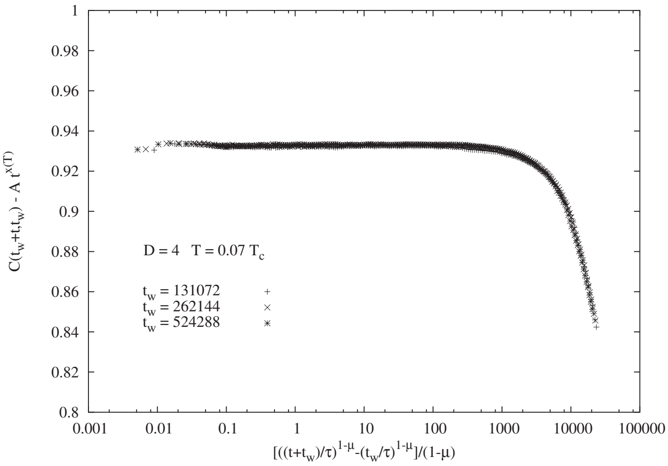

Fig. 3. Interrupted aging scaling for the intermediate time regime of the data shown in Figure 1. This time regime is characterized by an interrupted aging exponent $\mu = 0.27$ higher than the corresponding one seen in the asymptotic regime of Figure 2.

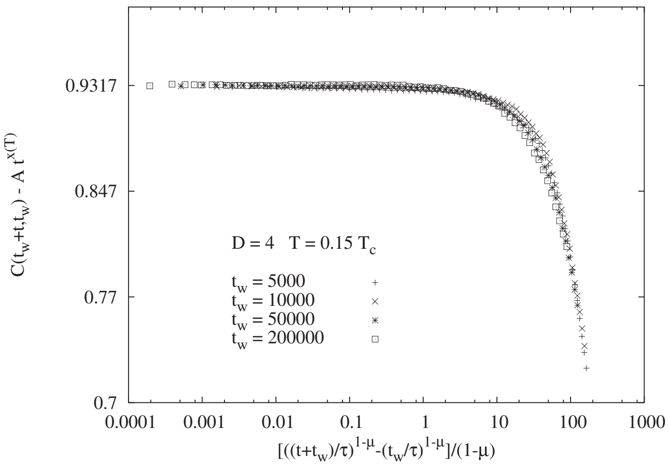

Fig. 4. Sub-aging scaling for the asymptotic regime of the d=4 Edwards-Anderson model at $T = 0.15T_c$ with linear size $L = 10$ and four different waiting times $t_w = 5000,\ 10\,000,\ 50\,000$ and $200\,000$. The fitting parameters of the stationary regime are $A = 0.03$ and $x(T) = 0.54$. In the long time regime the sub-aging exponent is $\mu = 0.7$.

produced by activation over low energy barriers. The scenario for this behavior is quite clear: after a long waiting time, the system diffuses further in a flat energy landscape surrounded by barriers of minimum height $\Delta E = 2$. At time scales of the order of $\exp(2/T)$ it can further relax by thermal activation over these low barriers. The next level is at $\tau \simeq \exp(4/T) \approx 2,56 \times 10^{12}$ and it is clearly unreachable. The simplicity of the landscape seen by the system at this temperature explains the behaviour similar to interrupted aging which is observed.

In Figure 4 we present the results of a simulation performed at a higher temperature (though still very low) $T = 0.15T_c$. At this temperature in the time range of the simulation the system is able to probe activation over barriers of height $\Delta E = 2$ and $\Delta E = 4$. The waiting times are $t_w = 5\,000$, $10\,000$, $50\,000$ and $200\,000$. The data can still be well collapsed for large values of t by a sub-aging scaling of the form (5), with an exponent $\mu \approx 0.7$. The sub-aging behaviour is getting weaker as the temperature is raised.

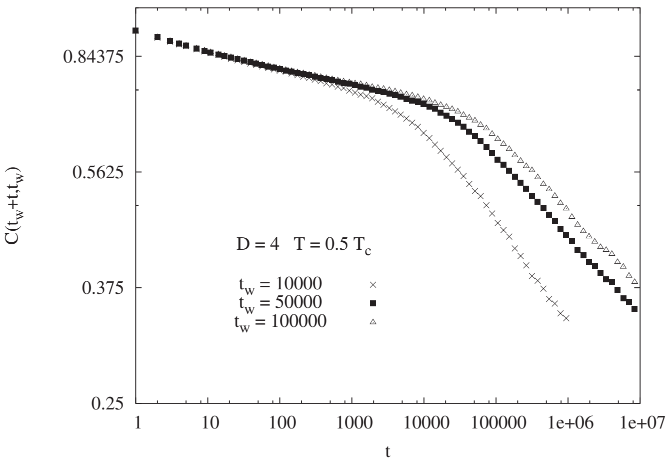

Fig. 5. Two-time autocorrelation function for the $\pm J$ spin glass in $d = 4$ with linear size $L = 10$ after a quench to $T = 0.5T_c$. The waiting times are $t_w = 10\,000, 50\,000$ and $10\,0000$.

3.2 Full aging dynamics

As the thermal energy is raised above the lowest lying levels the full ruggedness of the landscape should emerge and true aging dynamics should be restored. What happens when the thermal energy is enough to turn discrete energy levels undetectable? In Figure 5 we see the autocorrelations for a temperature $T = 0.5T_c$ for three waiting times $t_w = 10\,000$, $50\,000$, $100\,000$. A fit to the quasi-equilibrium region gives $x(T) = 0.1$ and $q = 0.62$, indicating a very slow relaxation. This is due to the increasing complexity of the phase space visited at this temperature. The best scaling form obtained for the aging regime is presented in Figure 6 and corresponds to the following logarithmic form:

$$C_{ag}(t_w + t, t_w) = \mathcal{C}\Big(\frac{\ln t}{\ln t_w}\Big)\cdot \qquad (8)$$

Note that there is still an intermediate regime which is not well described by this logarithmic scaling but the corresponding time window is too small to try a reasonable collapse there. On the other side, in the long time regime the logarithmic scaling (8) works remarkably well. It is worth noting that this form of the scaling function is compatible with dynamical ultrametricity (see [17]). This form is slightly different from that expected by a droplet like scenario, which takes the form $\ln(t + t_w)/\ln(t_w)$ and does not obey ultrametricity. This new evidence for a (weak) ultrametricity obtained directly from aging measurements is in agreement with recent results for the same model obtained from a quite different approach [18]. Nevertheless a simple aging scaling of the form $\mathcal{C}(t/t_w)$ also works well for the two largest waiting times and cannot be discarded although the overall behaviour of the ultrametric

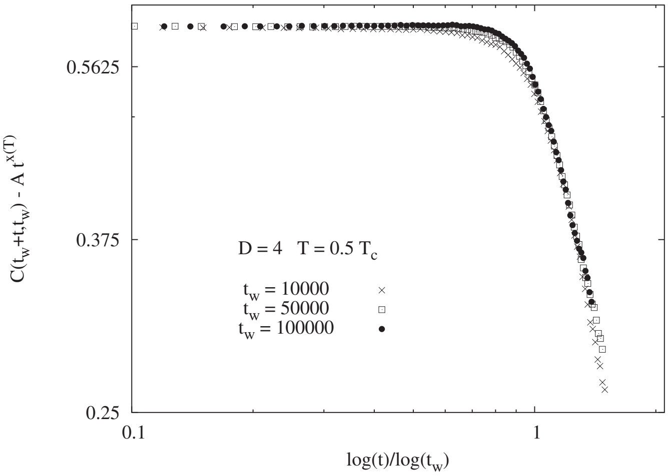

Fig. 6. Logarithmic aging scaling for the asymptotic time regime of the data shown in Figure 5. Fit parameters are $A = 0.3$ and $x(T) = 0.1$.

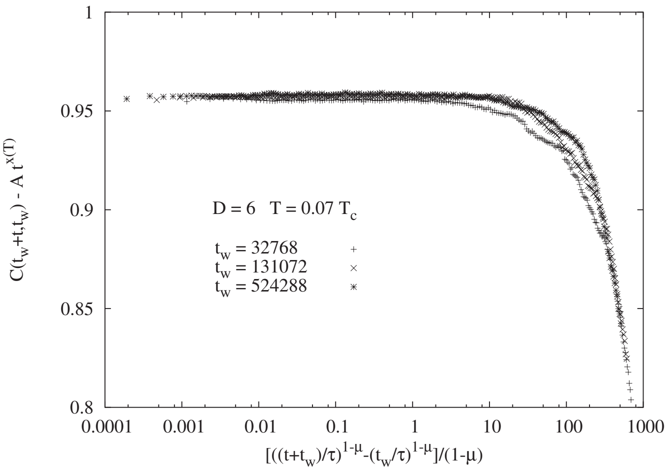

Fig. 7. Sub-aging scaling for two-time autocorrelation functions of the $d = 6$ Edwards-Anderson $\pm J$ spin glass at $T = 0.07T_c$. The fit parameters are $A = 0.027$, $x(T) = 0.55$ and $\mu = 0.65$.

scaling is better. We have also done simulations at temperature $T = 0.75T_c$ (not shown) and the results are compatible with the logarithmic scaling (8) although due to the strong noise at this rather high temperature the data is not so clean as at lower temperatures and the scaling of the curves is not so good. At high temperatures several time scales are mixed or superposed because of thermal noise and consequently it is difficult to obtain reliable scalings.

3.3 d = 6

We have done a similar analysis of the data obtained for the $d = 6$ Edwards-Anderson model. As this model is at the upper critical dimension we expected to see a behavior similar to the mean field or Sherrington-Kirkpatrick (SK) model [16,19,20]. The off equilibrium dynamics of the SK model is only poorly understood due to the complexity which emerges from the full replica symmetry breaking which is the central characteristic of that model. As a consequence of the complex ultrametric organization of time scales no simple scaling form can be found for the aging dynamics which might be described by a superposition of scaling regimes of the form [16]:

$$C_{ag}(t_w + t, t_w) = \sum_i \mathcal{C}_i\Big(\frac{h_i(t_w + t)}{h_i(t_w)}\Big)\cdot \qquad (9)$$

Nevertheless we have found a much simpler scenario which is in fact very similar to what is observed in $d = 3$ and $d = 4$. The transition to an SK like aging scenario seems to be very slow as the connectivity of the system grows.

All these simulations were carried out for systems with linear size $L = 5$. It is worth noting that the computational time required for simulating these systems increases as L^6, making it much more difficult to use larger values of L. The critical temperature of this model was estimated in [21,22] to be $T_c \simeq 3$. Here again we start by considering the very low temperature case $T = 0.07T_c$.

The overall behavior of the two-time autocorrelation function is similar to that described for dimension $d = 4$. In Figure 7 we show the best scaling obtained for the long time behavior of the autocorrelation after subtraction of the stationary part. The plots correspond to the three longest waiting times: $t_w = 2^{15}$, 2^{17} and 2^{19}. The fitting in the quasi-equilibrium regime yielded $x(T) = 0.55$ and $q \simeq 0.96$, similar to those found for $d = 4$ at the same temperature. The long time aging regime can be well collapsed with a sub-aging scaling of the form (5) with an exponent $\mu = 0.65$. It is also evident from the figure the presence of an intermediate regime which was not possible to collapse with scaling functions of the form (5) nor (6).

In Figure 8 we present the results of simulations at $T = 0.15T_c$. In this case all the data can be very well collapsed by a unique sub-aging scaling form (5) with $\mu = 0.97$. It is apparent that the dynamics of the model in $d = 6$ is slower than the corresponding dynamics in $d = 4$ at the lowest temperatures. Already at the very low temperature $T = 0.15T_c$ the $d = 6$ model is aging with a scaling behaviour very near simple aging observed in many other systems.

At $T = 0.5T_c$ the situation is also similar to what we found in $d = 4$. The best scaling of the whole data set is shown in Figure 9 where the ultrametric form (8) was used. Again the data in the long time aging regime is also compatible with a simple aging scenario.

In summary, as mentioned at the beginning of this section the $d = 6$ model presents an aging dynamics very similar to the $d = 4$ case at corresponding temperatures and it seems to be still far from the limit of the mean field system studied in [19,20].

4 Discussion and conclusion

In this paper we have presented an extensive numerical study of the out of equilibrium dynamics of spin glasses

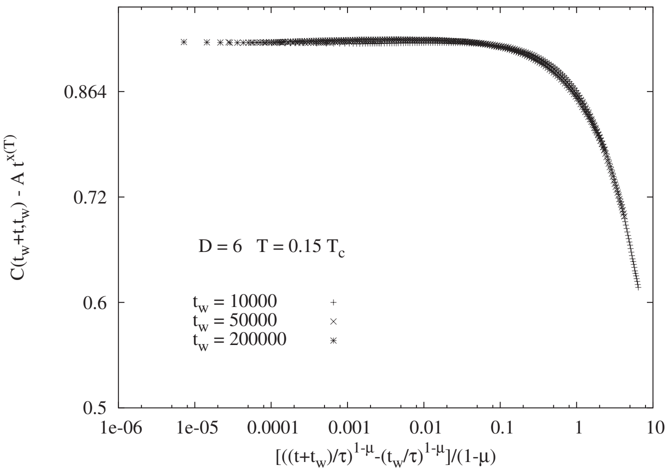

Fig. 8. Sub-aging scaling for the two-time autocorrelation functions for the $d = 6$ $\pm J$ spin glass at $T = 0.15T_c$. Fit parameters are $A = 0.04$, $x(T) = 0.29$ and $\mu = 0.97$.

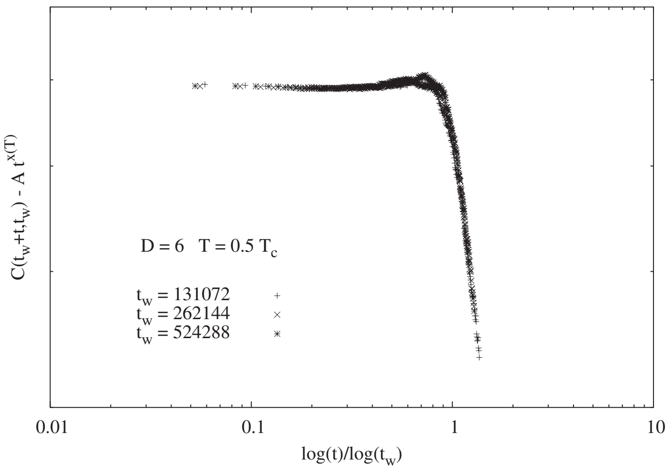

Fig. 9. Logarithmic scaling for the two-time autocorrelation functions of the $d = 6$ Edwards-Anderson spin glass at $T = 0.5T_c$. Fit parameters are $A = 0.34$ and $x(T) = 0.16$ and $q = 0.59$.

with discrete couplings defined on hypercubic lattices in 4 and 6 dimensions, for very low temperatures $T = 0.07T_c$, $T = 0.15T_c$ and $T = 0.5T_c$.

The relaxation behavior of the model with $\pm J$ couplings is different from that with continuous Gaussian couplings. While the observed aging dynamics of the Gaussian model is described in a wide range of temperatures and time scales by simple aging or weak interrupted aging scalings, the observed behavior of the $\pm J$ model seems to depart from that. At the lowest temperatures studied, that is for $T = 0.07T_c$ and $T = 0.15T_c$, the thermal energy is still not enough to permit activation over several scales and only the lowest lying energy levels are probed. As a consequence the system diffuses on a flat energy landscape surrounded by low barriers of height 2 or at most 4. It relaxes in this simple landscape presenting a rapid relaxation to stationarity similar to interrupted aging although the system if very far from equilibrium time scales. In this sense these relaxations are practically independent of t_w and true aging is restored only at higher temperatures. In order to see true aging at these very low temperatures we would have to wait for extremely long times. At $T = 0.5T_c$ both in $d = 4$ and $d = 6$ the full aging is restored, no more signs of strong sub-aging are seen but a slower logarithmic scaling is observed. A very weak sub-aging scenario ($\mu \leq 1$) is also compatible with the data. At this intermediate temperature range thermal energy is large compared with the low lying energy states which were important at the lower temperatures. Consequently the aging dynamics proceeds slowly but with the tendency of restoring ergodicity and equilibrium as t_w grows. We noted that for reasonably high temperatures $T = 0.5T_c$, and $T = 0.75T_c$ the best scaling was logarithmic but a simple aging scenario is slightly worse and cannot be discarded. The logarithmic aging observed at these temperatures points to a simple hierarchy of time scales with dynamical ultrametricity as observed in [18]. Nevertheless it is clear that it is very hard to decide which is the correct scaling form within the time scales presently available in computer simulations.

On the other hand, even for the model in $d = 6$, which is at the upper critical dimension, the qualitative form of the aging curves and the scaling functions are very different from those found in the SK model [19] and in the Hopfield model [20] which are known to present full replica symmetry breaking and a consequently full hierarchical or ultrametric organization of time scales [16]. In this respect it is worth citing recent work by Yoshino, Hukushima and Takayama [12] where they presented an extended version of the dynamical droplet theory. Two interesting new predictions are the presence of a new dynamical order parameter $q_D < q_{EA}$ which should be observed in a particular length/time scaling regime and the distinction between the times at which time translation invariance and the Fluctuation-Dissipation theorem are violated. The authors presented numerical results on the $d = 4$ $\pm J$ spin glass to support their findings but clearly more precise measurements are needed in order to test the theoretical predictions. In particular, scaling functions depend on the growth law of the coherence length $L(t)$ which permits to see the crossover between critical to activated dynamics. Unfortunately this information is still not available for the temperatures and time scales reached in our simulations which we think are necessary in order to clearly see the separation of time regimes during aging dynamics. We expect new interesting results to come from these kind of analysis in the near future.

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